

LABORATORY 9 – Active Filter Designs

Lab Goals

For this lab you will design, scale, construct, and test active filter circuits. You will compare the frequency performance of two different filters.

Definitions

Active Filter – filters constructed with op-amps. These filters can potentially exhibit power gain, i.e. have zero or even “positive” insertion loss (or gain).

Circuit Analysis

Consider the op-amp circuit in Fig. 10.1. Recalling the general formula that we derived for inverting amplifiers in the class, we can quickly get the voltage transfer function for this circuit:

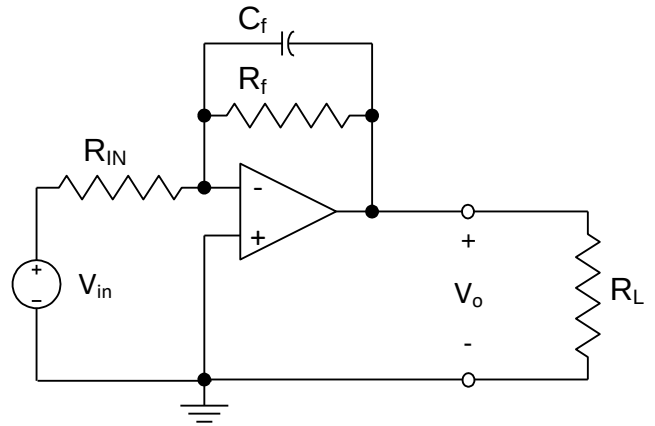


Figure 10.1 A simple active low pass filter.

$$\frac{\hat{V}_o}{\hat{V}_{in}} = -\frac{Z_f}{Z_i} = -\left(\frac{R_f / sC_f}{R_f + 1/sC_f} \right) / R_{in} = \frac{-R_f / R_{in}}{1 + sR_f C_f}.$$

Our normalized filter comes from selecting $C_f = 1$ F, $R_f = 1$ Ω , and $R_{in} = 2$ Ω , so

$$\frac{\hat{V}_o}{\hat{V}_{in}} = \frac{-1/2}{1+s} = \frac{-1/2}{1+j\omega}. \quad (10.1)$$

The magnitude of the transfer function is:

$$\left| \frac{\hat{V}_o}{\hat{V}_{in}} \right| = \frac{1/2}{\sqrt{1+\omega^2}}.$$

Thus, the circuit is a low-pass Butterworth filter. For low frequencies

$$\left| \hat{V}_o / \hat{V}_{in} \right| = -6 \text{ dB}.$$

Recall that this value is called the insertion loss. The active circuit does not have to have a negative insertion loss; R_{in} was chosen to be twice R_f only so the op-amp transfer function is identical to a 1st-order, low-pass, passive filter. Zero insertion loss is found by making $R_{in} = R_f$ and passband gain will occur for $R_{in} < R_f$. For high frequencies ($\omega \gg 1$), the circuit in Fig. 10.1 has

$$\left| \hat{V}_o / \hat{V}_{in} \right| \approx -6 - 20 \log_{10}(\omega) \text{ dB}$$

Thus, the roll-off is approximately 20dB/decade, indicating that this is indeed a 1st-order filter. Note that this filter is normalized for a 3dB frequency of 1 Rad/sec. The denormalization formulas from the previous lab (Eqs. 9.1) can be used to design a filter with a different corner frequency. Since ideal op-amps have zero output resistance, the filter load does not have to be matched to the source load. Instead, the resistance scaling factor in Eqs. 9.1 should be one that makes the resistors in the filter design be reasonable values for the op-amp circuit.

Consider the active filter shown in Fig. 10.2. This circuit is called a (single-ended) MFB filter, or multi-feedback filter, since there are two distinct feedback paths. To solve for the voltage transfer function, we write KCL at node N:

$$\hat{I}_{R_1} + \hat{I}_{R_2} + \hat{I}_{R_3} + \hat{I}_{C_1} = 0$$

Because the inverting terminal is at virtual ground, we can easily write those four currents in terms of $\hat{V}_{in}$, $\hat{V}_o$, and $\hat{V}_N$:

$$\begin{aligned} \hat{I}_{R_1} &= (\hat{V}_{in} - \hat{V}_N) / R_1 & \hat{I}_{R_2} &= -\hat{V}_N / R_2 \\ \hat{I}_{R_3} &= (\hat{V}_o - \hat{V}_N) / R_3 & \hat{I}_{C_1} &= -sC_1 \hat{V}_N \end{aligned}$$

But the current into the inverting terminal is zero, so

$$-\hat{V}_N / R_2 = \hat{I}_{R_2} = \hat{I}_{C_2} = sC_2 \hat{V}_o,$$

or

$$\hat{V}_N = -sR_2C_2 \hat{V}_o.$$

Therefore, we can combine all the equations and rearrange the terms to get the voltage transfer function:

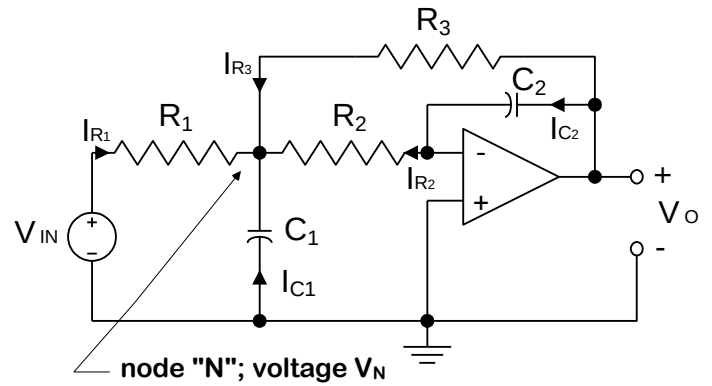


Figure 10.2 An active low pass filter circuit.

$$\begin{aligned}\frac{\hat{V}_{01}}{\hat{V}_{in_1}} &= \frac{-1}{R_1 / R_3 + sC_2 R_2 R_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + sC_1 \right)} \\ &= \frac{-1/(R_1 R_2 C_1 C_2)}{s^2 + s \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \frac{1}{C_1} + 1/(R_2 R_3 C_1 C_2)}.\end{aligned}$$

To get our normalized filter, we are going to choose $R_1 = R_2 = R_3 = 1 \Omega$, $C_1 = 3F$, and $C_2 = 1/3F$. Plugging these values in yields (note the inverting nature of the filter):

$$\frac{\hat{V}_{01}}{\hat{V}_{in_1}} = \frac{-1}{s^2 + s + 1} = \frac{-1}{(j\omega)^2 + j\omega + 1} \quad (10.2)$$

so

$$\left| \frac{\hat{V}_{01}}{\hat{V}_{in}} \right| = \frac{1}{\sqrt{(1-\omega^2)^2 + \omega^2}} = \frac{1}{\sqrt{1-\omega^2 + \omega^4}}.$$

A plot of the magnitude of this transfer function is given in Fig.10.3. Note that the response is not maximally flat. In fact, the response has a maximum that can be found by setting the derivative of the transfer function to zero. The result is

$$\omega_{\max} = 1/\sqrt{2}$$

and

$$\left| \frac{\hat{V}_{out}}{\hat{V}_{in}}(\omega_{\max}) \right| = 2/\sqrt{3}.$$

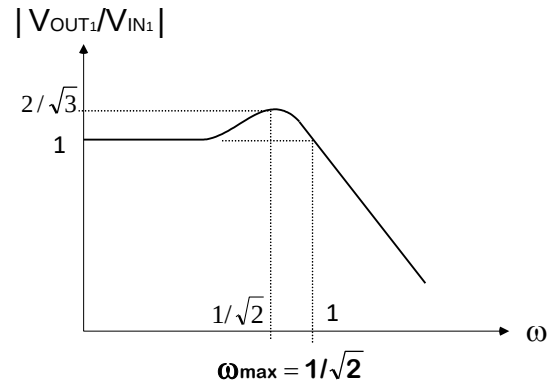


Figure 10.3 Frequency response for the circuit in Fig. 10.2.

Finding the gain in dB yields:

$$\left| \hat{V}_{01} / \hat{V}_{in_1} \right| (dB) = 20 \log_{10} \frac{1}{\sqrt{1-\omega^2 + \omega^4}} = -10 \log_{10} (1-\omega^2 + \omega^4).$$

For high frequencies

$$\left| \hat{V}_{01} / \hat{V}_{in_1} \right| (dB) = -40 \log_{10} \omega,$$

so the roll off is 40 dB/decade and this is a second order low-pass circuit. This is not surprising because there are two capacitors, one in parallel with the output and one in parallel with the input.

To scale this result to an unnormalized frequency and impedance, you use the same scaling rules that were spelled out for passive filters.

To arrive at a 3rd-order Butterworth filter, we are going to cascade a 1st-order op-amp stage on the end of the 2nd-order circuit we just analyzed. The net circuit is shown in Fig. 10.4.

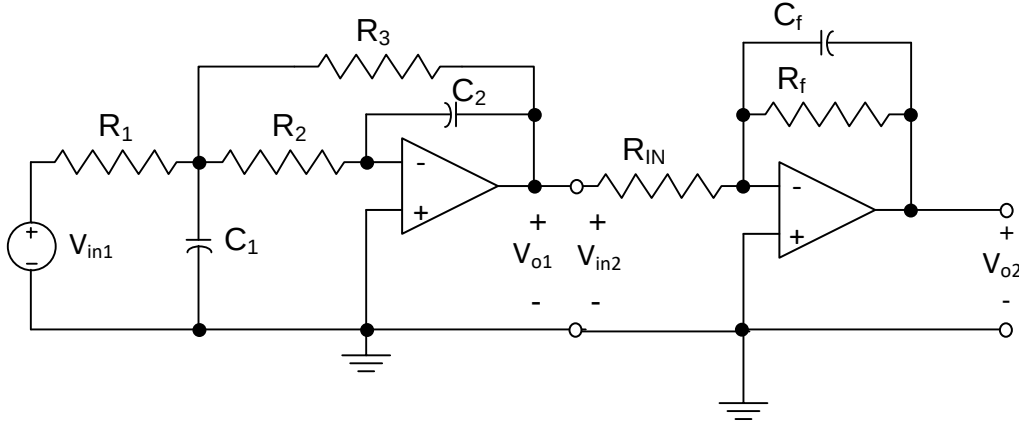


Figure 10.4 An 3rd-order low pass filter circuit.

The net transfer function when these two op-amp circuits are cascaded in series is just the product of Eqs. 10.1 and 10.2:

$$\frac{\hat{V}_{o_2}}{\hat{V}_{in_1}} = \frac{\hat{V}_{o_2}}{\hat{V}_{in_2}} \frac{\hat{V}_{in_2}}{\hat{V}_{in_1}} = \frac{-1/2}{(1+j\omega)} \cdot \frac{-1}{(1+j\omega+(j\omega)^2)} \quad (10.3)$$

or

$$\frac{\hat{V}_{o_2}}{\hat{V}_{in_1}} = \frac{1/2}{1-2\omega^2+j\omega(2-\omega^2)}.$$

This is the same expression we had for the 3rd-order passive circuit in the previous lab, so once again the magnitude of the gain is

$$\left| \frac{\hat{V}_{o_2}}{\hat{V}_{in_1}} \right| = \frac{1/2}{\sqrt{1+\omega^6}}. \quad (10.4)$$

The disadvantage of the active filter is that it requires op-amps, which in turn require two power supplies in order to function properly. A passive filter can be constructed, placed in a tiny box with the appropriate connectors, and inserted in a signal line and function properly without any concerns or maintenance.

On the other hand, the active circuit has many advantages. First, it requires no inductors, which are often heavy, not readily available off-the-shelf in a variety of values, and can interact with other nearby inductors. Also, the active filter gain is fairly independent of the load impedance. This is not true for passive circuits as you may have seen in the previous lab. Finally, an active filter can actually have an insertion gain, rather than an insertion loss, by choosing proper values for the resistors.

Note that in this section we only analyzed two very specific low-pass filter designs. There are many types of filters and it is beyond the scope of the course to have a thorough discussion of the general design procedure. However, a few more brief comments are appropriate before we end this section. If you need further information, there are several books that give normalized circuit configurations for low pass, band pass, band notch, and high pass Butterworth filters of many different orders. Also, there is free software on the web that can be used to design active filters. One example is FilterPro Desktop by Texas Instruments (version 3.1 in 2011), which can certainly be used to design the circuits for this lab.

Active filter designs for higher than 2nd-order are generally done with cascaded amplifiers of 1st and 2nd-order. Another popular second order design is the Sallen-Key filter shown in Fig. 10.5. This filter is nice because it is a non-inverting configuration with a separately-controllable gain. The MFB achieves gain and filtering with fewer components, but is inverting.

Figure 10.6 shows 1st-order non-inverting low pass and high pass topologies. Figure 10.7 shows additional MFB topologies for high pass and band pass filters. An MFB band-notch filter requires two op-amps and is not presented here. None of the component values are listed in Figs. 10.5 – 10.7. You need to use a design code such as FilterPro Desktop to design op-amps for this lab. In addition to the frequency information, designs depend on the pass-band characteristics. Popular filter designs include Butterworth (maximally flat), Chebyshev (steeper roll-off), and Bessel (flat group delay).

G. Helpful Hints

1. Build and test any 3rd-order (or higher) active filter in stages, i.e. make sure the first op-amp circuit works before building the second op-amp circuit.

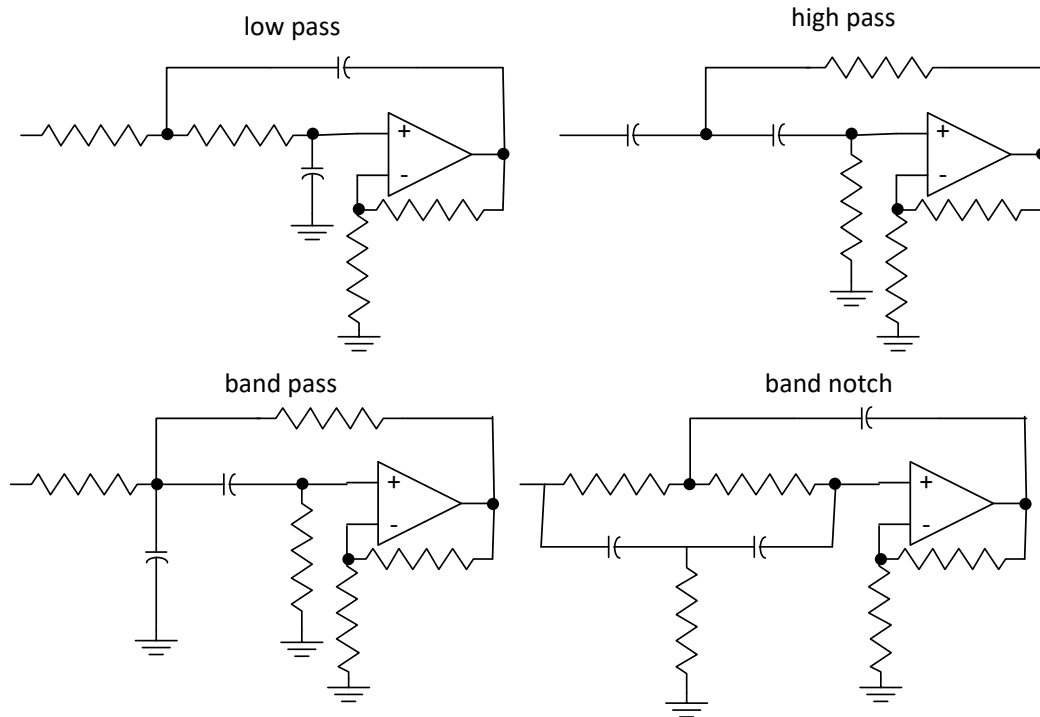


Figure 10.5. 2nd-order Sallen-Key Filters.

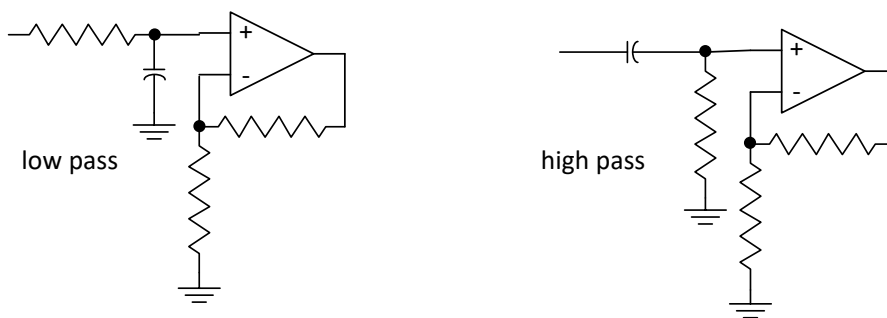


Figure 10.6. Non-inverting 1st-order Filters.

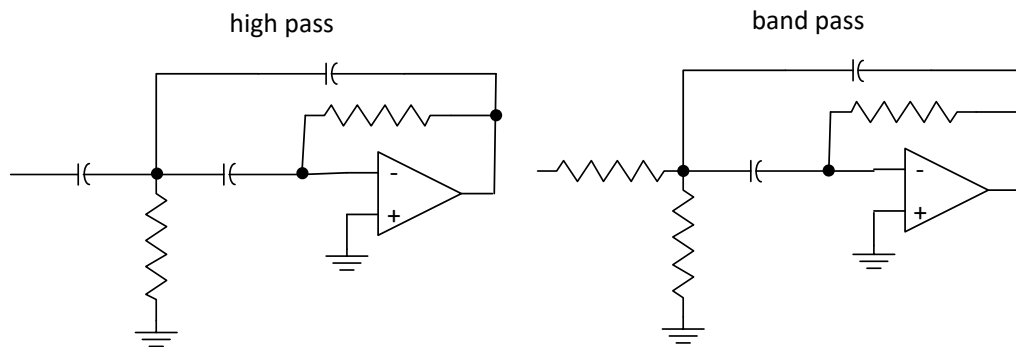


Figure 10.7. More 2nd-order MFB Filters.

Laboratory 9 Description - Active Filter Designs

Objective:

To design, build, analyze, compare and contrast the performance of various filter circuits.

Pre-lab preparation

Part I – Design a low-pass, second order, Sallen-Key, Butterworth filter with a corner frequency of about 10 kHz and a load resistance of about 1 k Ω . The gain should be 0 dB.

1. Draw the circuit diagram.
2. Use PSpice to simulate the circuit performance. Plot the output voltage over a suitable range of frequencies. Plot both the magnitude and phase of the output. Assume the input voltage is 4V peak-peak.

Part II – Second Filter

3. Design a high-pass, second order, Sallen-Key, Butterworth filter with a corner frequency of about 5 kHz and a load resistance of about 1 k Ω . The gain should be +3dB.
4. Draw the circuit diagram.
5. Use PSpice to simulate the circuit performance. Plot the output voltage over a suitable range of frequencies. Plot both the magnitude and phase of the output. Assume the input voltage is 4 V peak-peak.

Part III –Filter Combination

6. In PSpice, connect the output of the first filter to the input of the second filter and simulate the circuit performance. Plot the output voltage (of the second filter) over a suitable range of frequencies. Plot both the magnitude and phase of the output. Assume the input voltage (to the first filter) is 2 V peak-peak.

Experimental procedure:

Part 0 – Preparation

1. Measure all component values to be used in the filters.

Part I – First Filter

2. Use 5 V (plus and minus) for the op-amp power.
3. Construct the first filter circuit. Set the input voltage to 2 V peak-peak.

4. Measure the relative magnitude and phase of the output voltage from 10 Hz to 200 kHz. Find the 3 dB point and plot the input and output voltages.
5. DO NOT DISASSEMBLE THIS OP-AMP CIRCUIT!!!

Part II – Second Filter

6. Construct the second filter circuit. Set the input voltage to 2 V peak-peak.
7. Measure the relative magnitude and phase of the output voltage from 10 Hz to 200 kHz. Find the 3 dB point and plot the input and output voltages.

Part III – Combined Filters

8. Connect the output of the first filter to the input of the second filter. For this new, combined filter, measure the relative magnitude and phase of the output voltage from 10 Hz to 200 kHz. Find the 3 dB points and plot the input and output voltages.

Post-lab analysis:

Generate a lab report “following” the sample report available in Appendix A. Mention any difficulties encountered during the lab. Describe any results that were unexpected and try to account for the origin of these results (i.e. explain what happened). In ADDITION, answer the following questions:

Part I – for the first filter:

1. Compare the measured performance with the simulation results for the filter. Discuss both the relative amplitude and phase of the output qualitatively.
 - 1.1. What is the measured pass-band gain of the circuit?
 - 1.2. What is the corner frequency of the circuit?
 - 1.3. What is the measured roll-off (in dB/decade) far away from the 3 dB point of the circuit?
2. Plot the frequency response (**magnitude and phase!!!**) on a log-log graph for the frequency sweep mentioned in the experimental section above.

Part II – for the second filter:

3. Compare the measured performance with the simulation results for the filter. Discuss both the relative amplitude and phase of the output qualitatively.
 - 3.1. What is the measured pass-band gain of the circuit?
 - 3.2. What is the corner frequency of the circuit?
 - 3.3. What is the measured roll-off (in dB/decade) far away from the 3 dB point of the circuit?

4. Plot the frequency response (**magnitude and phase!!!**) on a log-log graph for the frequency sweep mentioned in the experimental section above.

Part III – for the combined filter:

5. Compare the measured performance with the simulation results for the combination. Discuss both the relative amplitude and phase of the output qualitatively.
 - 5.1. What is the measured pass-band gain of the circuit?
 - 5.2. What are the corner frequencies of the circuit?
 - 5.3. What is the measured roll-off (in dB/decade) far away from the 3 dB point of the circuit?
6. Plot the frequency response (**magnitude and phase!!!**) on a log-log graph for the frequency sweep mentioned in the experimental section above.

Part IV – active/passive comparison:

7. Which filters performed better: the passive filters from lab 8 or the active filters from this lab? Explain your answer!
8. Which filter would you choose to build (active or passive) for a long-term application in a remote location? Explain your answer!